**Postulates**

**Postulate 1: The Distance Postulate**

To every pair of different points there corresponds a unique positive number.

**Postulate 2: The Ruler Postulate**

The points of a line can be placed in correspondence with the real numbers in such a way that…

1.) To every point of the line there corresponds exactly one real number.

2.) To every real number there corresponds exactly one point of the line; and

3.) The distance between any two points is the absolute value of the difference of the corresponding numbers.

**Postulate 3: The Ruler Placement Postulate**

Given two points P and Q of a line, the coordinate system can be chosen in such a way that the coordinate of P is zero and the coordinate of Q is positive.

**Postulate 4: Line Postulate**

For every two points there is exactly one line that contains both points.

**Postulate 5**

Every plane contains at least three non – collinear points. Space contains at least four non – coplanar points.

**Postulate 6**

If two points of a line lie in a plane, then the line lies in the same plane.

**Postulate 7: The Plane Postulate**

Any three points lie in at least one plane, and any three non – collinear points lie in exactly one plane.

**Postulate 8**

If two different planes intersect, then their intersection is a line.

**Plane Separation Postulate**

Given a line and a plane containing it. The points of the plane that do not lie on the line form two sets such that:

1.) Each of the set is convex set, and

2.) If P is in one of the sets and Q is in the other, then the segment PQ intersects the line.

**Space Separation Postulate**

The points of space that do not lie in each plane form two sets, suchthat:

1.) Each of the set is convex, and

2.) If P is in one of the sets and Q is in the other, then the segment PQ intersects the plane.

**SSS Postulate (Side - Side - Side)**

If the sides of one triangle are congruent to the sides of a second triangle, then the triangles are congruent.

**SAS Postulate (Side - Included Angle - Side)**

If two sides and the INCLUDED angle of one triangle are congruent to two sides and the INCLUDED angle of another triangle, then the triangles are congruent.

**ASA Postulate (Angle - Included Side - Angle)**

If two angles and the INCLUDED side of one triangle are congruent to two angles and the INCLUDED side of another triangle, then the triangles are congruent.

**AAS Postulate (Angle - Angle - Side)**

If two angles and a NON-INCLUDED side of one triangle are congruent to the corresponding two angles and side of a second triangle, the two triangles are congruent.

**Theorems**

**Theorem 1**

If A, B, and C are three different points of the same line, then exactly one of them is between the other two.

**The Point Plotting Theorem**

Let AB be a ray, and let x be a positive number. Then there is exactly one point P of AB such that AP = x.

**Theorem 3**

Every segment has exactly one midpoint.

**Theorem 4**

If two different lines intersect, their intersection contains only one point.

**Theorem 5**

If a line intersects a plane not containing it, then the intersection contains only one point.

**Theorem 6**

Given a line and a point not on the line, there is exactly one plane containing both.

**Theorem 7**

Given two intersecting lines, there is exactly one plane containing both.

**Theorem 8**

Every right angle has measure 90, and every angle with measure 90 is a right angle.

**Theorem 9**

Congruence between angles is an equivalence relation.

**Theorem 10**

If two angles are complementary, then both are acute.

**Theorem 11**

Any two right angles are congruent.

**Theorem 12**

If two angles are both congruent and supplementary, then each is a right angle.

**The Supplement Theorem**

Supplements of congruent angles are congruent.

**The Complement Theorem**

Complements of congruent angles are congruent.

**The Vertical Angle Theorem (VAT)**

Vertical angles are congruent.

**Theorem 16**

If two intersecting lines form one right angle, then they form four right angles.

**Theorem 17**

The perpendicular bisectors of the sides of a triangle are concurrent at a point equidistant from the vertices.

**Theorem 18**

The bisectors of the angles of a triangle are concurrent at a point equidistant from the sides.

**Theorem 19**

The medians of a triangle intersect in a point that is two thirds of the distance from each vertex to the midpoint of the opposite side.

**The Perpendicular Bisector Theorem (PBT)**

The perpendicular bisector of a segment, in a plane, is the set of all points of the plane that are

equidistant from the end points of the segment.

If line L is the perpendicular bisector of then:

1.) If P is on L, then PA = PB, and

2.) If PA = PB, then P is on L.

**Converse of Perpendicular Bisector Theorem (CPBT**

Given a segment and a line L in the same plane. If two points of L are each equidistant from A and B, then L is the perpendicular bisector of.

**Theorem 22/23**

Through a given external point there is at least/most one line to a given line.

**The Exterior Angle Theorem (EAT)**

An exterior angle of a triangle is greater than each of its remote interior angles.

**Theorem Related to EAT**

The measure of an exterior angle of a triangle is equal to the sum of the measures of the remote interior angles, from the given figure above: If ∠A and ∠B are the remote interior angle of exterior angle ∠BCD, then m∠BCD = m∠A + m∠B.

**The SAA Theorem**

Every SAA correspondence is a congruence.

**The Hypotenuse – Leg Theorem**

Given a correspondence between two right triangles. If the hypotenuse and one leg of the triangles are congruent to the corresponding parts of the second triangle, then the correspondence is a congruence.

**Theorem 28**

If two sides of a triangle are not congruent, then the angles opposite them are not congruent, and the larger angle is opposite the longer side.

**Theorem 29**

If two angles of a triangle are not congruent, then the sides opposite them are not congruent, and the longer side is opposite the larger angle.

**The First Minimum Theorem**

The shortest segment joining a point to a line is the perpendicular segment.

**The Triangle Inequality Theorem**

The sum of any two sides of a triangle is greater than the length of the third side.

**The Hinge Theorem**

If two sides of one triangle are congruent, respectively, to two sides of a second triangle, and the included angle of the first triangle is larger than the included angle of the second, then the third side of the triangle is longer than the third side of the second.

**The Converse Hinge Theorem**

If two sides of one triangle are congruent respectively, to two sides of a second triangle, and the third side of the first triangle is longer than the third side of the second, then the included angle of the first triangle is larger than the included angle of the second.

**Theorem: Existence of Parallels**

Let L be a line and let P be a point not on L. Then there is at least one line through P, parallel to L.

**Theorem 35**

If two lines are cut by a transversal, and one pair of alternate interior angles are congruent, then the other pair of alternate interior angles are also congruent.

**Definitions**

**Definition of Right Angle**

If the angles in a linear pair have the same measure, then each of themis called a right angle.

**Definition of Segment**

A segment is the part of a line consisting of two endpoints and all points between them.

**Definition of Ray**

A ray is the part of a line consisting of one endpoint and all the points of the line on one side of the endpoint.

**Opposite rays** are two collinear rays with the same endpoint. Opposite rays always form a line.

**Definition of Betweenness**

B is between A and C if:

1.) A, B, and C are different points of the same line, and

2.) AB + BC = AC.

When B is between A and C, we write A – B – C or C – B – A (read as B is between A and C or B is between C and A.

**Definition of Midpoint**

A point B is called a midpoint of a segment AC if B is between A and C and AB = BC.

**Definition of Bisector**

The midpoint of a segment is said to bisect the segment. The midpoint of a segment AB, or any line, plane, ray, or segment which contains the midpoint and does not contain AB is called a bisector of AB.

**Angle Bisector of Triangle**

It lies in the ray which bisects an angle of a triangle, and its end points are the vertex of this angle and a point of the opposite side.

**Median of Triangle**

A segment whose endpoints are a vertex of the triangle and the midpoint of the opposite side.

**Altitude of Triangle**

A segment, perpendicular from a vertex of the triangle to the line containing the opposite side.

**Perpendicular Bisector of the Sides of a triangle**

A segment perpendicular to the side of a triangle at its midpoint.

**Definition of Perpendicular Bisector**

In a given plane, the perpendicular bisector of a segment is a line which is perpendicular to the segment at its midpoint.

**Definition of Exterior Angle**

If C is between A and D, then ∠BCD is an exterior angle of ∆ABC.

**Definition of Remote Interior Angle**

∠A and ∠B of ∆ABC are called the remote interior angles of the exterior angles of the angles ∠BCD and ∠ACE.

**Definition of Quadrilateral**

Let A, b, C, and D be four points of the same plane. If no three of these points are

collinear, and the segments AB̅̅̅̅̅, BC̅̅̅̅̅, CD̅̅̅̅, and DA̅̅̅̅ intersect only at their endpoints,

ten the union of these four segments is called a **quadrilateral**.

**Definition of Parallelogram**

A parallelogram is a quadrilateral in which both pairs of opposite sides are parallel.

**Definition of Rhombus**

A rhombus is a parallelogram all of whose sides are congruent.

**Definition of Rectangle**

A rectangle is a parallelogram all of whose angles are right angles.

**Definition of Square**

A square is a rectangle all of whose sides are congruent.

**Definition of Trapezoid**

A trapezoid is a quadrilateral with exactly one pair of parallel sides. The segment joining the midpoints of the nonparallel sides is called the **median.**

**Definition of Kite**

A kite is a quadrilateral in which exactly one diagonal is the perpendicular bisector of

the other.

**Definition of Diagonal of a Quadrilateral**

A diagonal of a quadrilateral is a segment joining two nonconsecutive vertices.

**Other Definitions**

· A set of points is **collinear** if there is a line which contains all the points of the set.

· A set of points is **coplanar** if there is a plane which contains all points of the set.

· **Noncollinear points** – points that do not lie on the same line.

· **Noncoplanar points** – points that do not lie on the same plane.

· A set of all points is called **space**.

· A set M is called **convex** if for every two points P and Q of the set, the entire segment PQ lies in M.

· The distance between a line and an external point is the length of the perpendicular segment from the point to the line. The distance between a line and a point on the line is defined to be zero.

**Corollary**

**Corollary 1**

No triangle has two right angles.

**Corollary of the Exterior Angle Theorem**

If a triangle has one right angle, then its other angles are acute.

**Corollary Theorem to Sum of Measures of a Triangle**

Given a correspondence between two triangles. If two pairs of corresponding angles are

congruent, then the third pair of corresponding angles are also congruent.

**Corollary Theorem to Sum of Measures of a Triangle**

For any triangle, the measure of an exterior angle is the sum of the measures of the two

remote interior angles.

**The PCA Corollary**

If two parallel lines are cut by a transversal, each pair of corresponding angles are congruent.

**Conditions which guarantee Parallelism**

**Theorem 1**

In a plane, two lines are parallel if they are both perpendicular to the same line.

**The AIP Theorem**

Given two lines cut by transversal. If a pair of alternate interior angles are congruent, then the lines are parallel.

**Theorem 3**

Given two lines cut by a transversal. If a pair of corresponding angles are congruent, then a pair of alternate interior angles are congruent.

**The CAP Theorem**

Given two lines cut by a transversal. If a pair of a corresponding angles are congruent, then the lines are parallel.

**Theorem 5**

Given two lines cut by a transversal. If a pair of interior angles on the same side of transversal are supplementary, then the lines are parallel.

**The Parallel Postulate**

Through a given external point there is only one parallel to a given line.

**The PAI Theorem**

If two parallel lines are cut by a transversal, then alternate interior angles are congruent.

**Theorem 8**

If two parallel lines are cut by a transversal, the interior angles on the same side of the transversal are supplementary.

**Theorem 9**

In a plane, if a line intersects one of two parallel lines in only one point, then it intersects the other.

**Theorem 10**

In a plane, if two lines are each parallel to a third line, then they are parallel to each other.

**Theorem 11**

In a plane, if a line is perpendicular to one of two parallel lines it is perpendicular to the other.

**Theorem 12**

For every triangle, the sum of the measures of the angles is 180.

**Theorems Concerning Quadrilaterals**

**Angle Sum Property of Quadrilateral**

In a convex quadrilateral, the sum of the measure of the angles is 360.

**Theorem 1**

Each diagonal separates a parallelogram into two congruent triangles.

**Theorem 2**

In a parallelogram, any two opposite sides are congruent.

**Theorem 3**

If two lines are parallel, then all points of each line are equidistant from the other line.

**Theorem 4**

In a parallelogram, any two opposite angles are congruent.

### **Theorem 5**

In a parallelogram, any two consecutive angles are supplementary.

### **Theorem 6**

The diagonals of a parallelogram bisect each other.

**Theorem 7**

The diagonals of a rectangle are congruent.

**Theorem 8**

In rhombus, the diagonals are perpendicular to one another.

**Theorem 9**

The diagonals of a rhombus bisect the angles of the rhombus.

**Midline Theorem (might not concern quadrilaterals but proving requires one)**

If a line segment’s endpoints are the midpoints of two sides of a triangle, then the remaining side is parallel to the line segment and the line segment is half as long as the remaining side.

**Theorem 10**

If 2 sides of a quadrilateral are parallel and congruent, then the quadrilateral is a parallelogram.

**Theorem 11**

If two opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

**Theorem 12**

If two opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

**Theorem 13**

If a parallelogram has one right angle, then the parallelogram is a rectangle.

**Theorem 14**

If the diagonals of a parallelogram bisect each other and are perpendicular, the parallelogram is a rhombus.

**Theorem 15**

The median of a trapezoid is parallel to the bases and its measure is half the sum of the measure of the bases.

**Formulas for Finding the Perimeter of a Quadrilateral**

|  |  |
| --- | --- |
| **Types of Quadrilaterals** | **Formula for finding the Perimeter** |
| Parallelogram | P= 2(Base+Side) |
| Square | P=4(Side) |
| Rectangle | P=2(length+width) |
| Rhombus | P=4(Side) |